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# Wavelet Approach to Chaotic Forecasting of Stock Movement

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**ABSTRACT**
A chaotic method is employed to forecast a near future of uncertain phenomena. The method makes it possible by restructuring an attractor of given time-series data in a multi-dimensional space through Takens' embedding theory. However, many economical time-series data are not sufficiently chaotic. In other words, it is hard to forecast the future trend of such economical data on the basis of chaotic theory. In this paper, time-series data are divided into wave components using wavelet transform. It is shown that some divided components of time-series data show much more chaotic in the sense of correlation dimension than the original time-series data. The highly chaotic nature of the divided component enables us to precisely forecast the value or the movement of the time-series data in a near future. The up and down movement of TOPICS value is shown as highly predicted by this method as 70%.

**Keywords**: Chaos theory, Short-term forecasting, Wavelet transform.

**JEL codes**:

## I. INTRODUCTION

The chaotic short-term forecasting method [1], Matsuba (1992); [2], Nagashima, Nagai and Ogiwara (1990) based on time-series data enables us to know a value, which we could not predict before. Nevertheless, it is still difficult to definitely forecast a value even in near future because many kinds of data are less chaotic. Even though such data are less chaotic, it is possible to abstract and pull out the partially chaotic portion out of the data [3], Matsumoto and Watada (1998); [4], Matsumoto and Watada (1998); [5], Matsumoto and Watada (2001).

In this research, wavelet transform [6], Chui (1992) is employed to take chaotic portions out of the original time-series data and we can find the more highly chaotic component out of the original data by measuring these correlated dimension. Once we can successfully find the highly chaotic portion out of the original data, it enables us to improve the forecasting precision by the wavelet transformation.

The correlation dimension [7], Brock (1986); [8], Scheinkman and LeBaron (1989) of the divided components should be measured smaller than the one of the original data, if the divided components are more highly chaotic than the original data.

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## II. CHAOTIC APPROACH AND FORECASTING

### A. Chaos Theory

Although the chaos in science means a disturbed state, it doesn't mean so large a disturbed state, but does a medium-sized disturbed phenomenon, which changes irregularly along a time in a sense. In other words, it means an irregularity of a changing phenomenon that is controlled by relatively simple rules or a simple structure. One typical example of a chaos system is a logistic mapping as shown in Fig.1. The logistic mapping can be defined by a very simple relation. The resulted state seems to show very random movement on a graph as shown in Fig.2.

**Figure 1. Logistic Function**

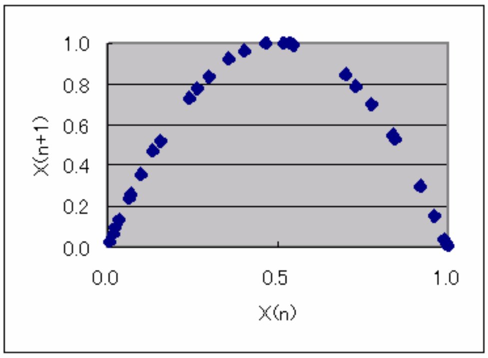



**Figure 2. Logistic Map**

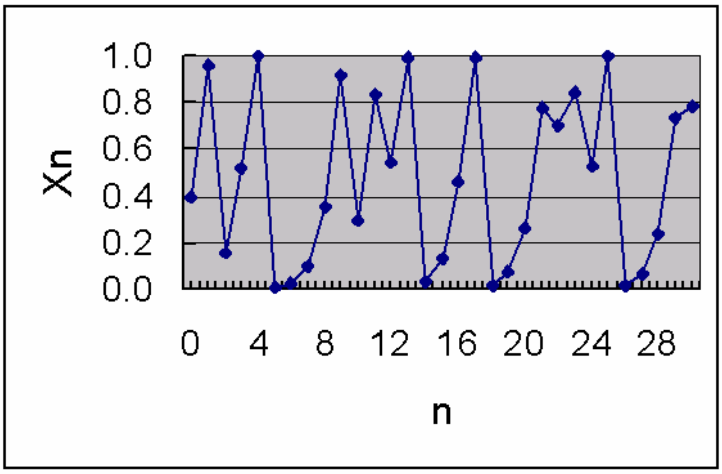
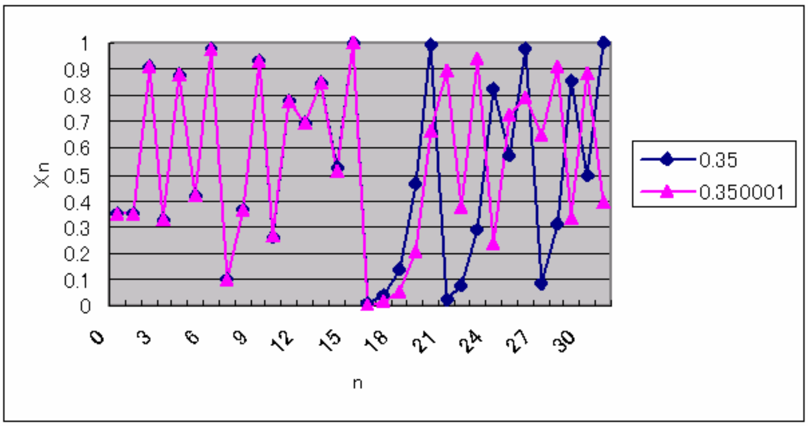



**Figure 3. Initial State of Logistic Function**



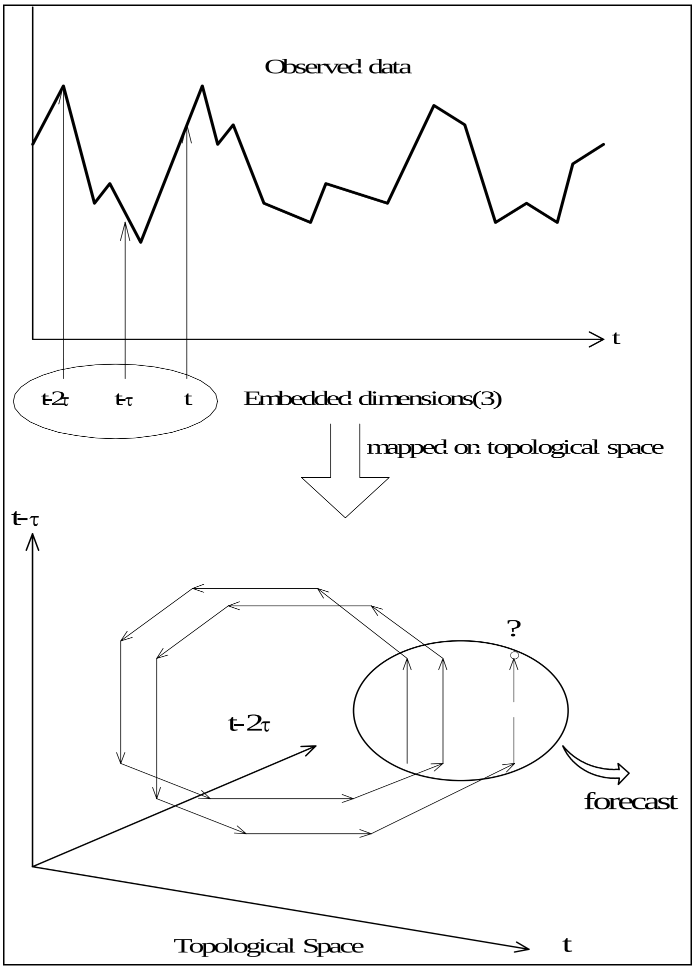

Especially, it is well known that the chaos system is sensitive to an initial state [9], Serizawa (1993). When the initial state is changed a little, the mapping by a chaotic mechanism shows very different trajectory beyond a short term as illustrated in Fig.3, even if it follows the same trajectory in the initial states. This phenomenon is called “sensitivity to an initial state”. Because of the “sensitivity to an initial state”, it is not appropriate to employ the chaos method in forecasting a value in long-term future. That is, the chaotic method enables us to predict only the state in near future, which is sufficiently influenced by the present state.

**B. Forecasting by Chaotic Method**

The general objective to employ the chaotic method in forecasting is 1) to find a deterministic structure in given time-series data and 2) to predict a value in such a near future from a certain point using this structure on that the present state can sufficiently influence. This chaotic method enables us to forecast the near future with high precision using time-series data which show very unpredictable and unperiodical change. This forecasting bases on the Takens' embedding theory [10], Takens (1981) which tells us that it is possible to restructure the trajectory of a dynamic system in a high dimensional space by using only the information (that is, time-series data) of partial component dimensions (variables).

Using time-series data $x(t)$, let us define vector $Z(t)$ as follows:

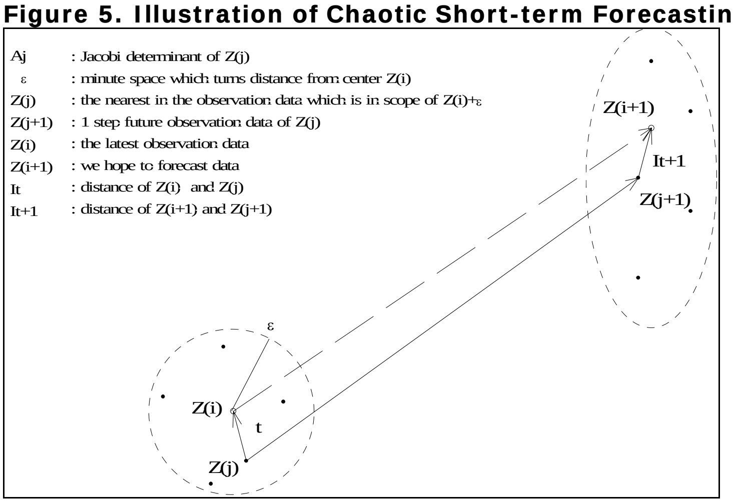
**Figure 4. $Z(i)$ mapped into an $n$ dimensional topological space**



$$Z(t) = (x(t), x(t-\tau), x(t-2\tau), \cdots, x(t-(n-1)\tau)) \quad (1)$$

where $\tau$ denotes an arbitrary constant time interval. The vector $Z(t)$ shows one point in an $n$ dimensional space (Data Space). Therefore, changing $t$ generates a trajectory in the $n$ dimensional data space. When $n$ is sufficiently large, this trajectory shows a smoothly changed one of the high dimensional dynamic system. That is, if the dynamic system has some attractor, the attractor obtained from the original one should come out on the data space. In other words, the original attractor of the dynamic system can be embedded in the $n$ dimensional topological space. Number $n$ is named an embedded dimension. Denoting the dimension of the original dynamic system by $m$, it can be proved that this dimension $n$ is sufficiently large if $n$ holds the following:

**Figure 5. Illustration of Chaotic Short-term Forecasting**




$$n \geq 2m + 1 \quad (2)$$

Equation (2) is a sufficient condition on the embedded dimension. It is required to employ data with more than $3m+1$ to $4m+1$ samples within a certain time length in short-term forecasting.

Next, let us illustrate the deterministic structure using a restructured trajectory. There are several methods. Figure 4 shows short-term forecasting using the chaotic method that embeds discrete time-series data with equal time interval $\tau$ in embedded dimension $n = 3$. Observed discrete time-series samples can be mapped into a topological space with an embedded 3-dimensional space as shown in Figure 4. As a result, the mapped vector is denoted in the following:

$$Z(t) = (x(t), x(t-\tau), x(t-2\tau)) \quad (3)$$

Let $Z(i)$ denote a 3-dimensional vector that maps observed data including the most recent time into a topological space. Figure 5 illustrates the plotted figure of data which are mapped around the neighborhood of $Z(i)$ in the 3-dimensional space.

These data in the neighborhood of $Z(i)$ are ones observed in the past. The trajectory of $Z(i+1)$ at one step future has been observed as shown in Figure 5. These relations enable us to forecast behavior $Z(i+1)$ in near future. The future trajectory $x(i+1)$ of the given time-series data ( $x(i)$ , $x(i-1)$ ,...) can be calculated as in the following procedure.

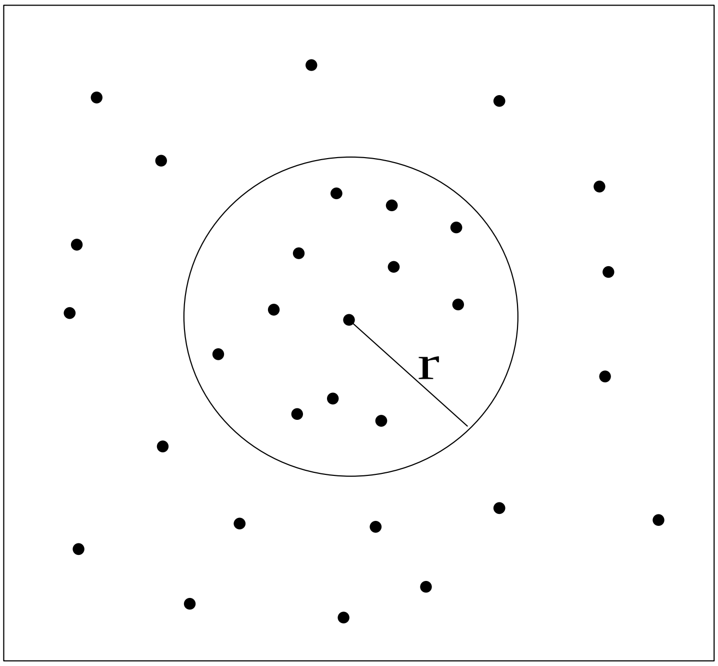
The forecasting procedure:
STEP1: decide the nearest points $Z(j)$ included in the neighborhood with diameter $r$ from $Z(i)$,
STEP2: calculate the distance $I_{t+1}$ between $Z(i+1)$ and $Z(j+1)$ using the Jacobi matrix $A_j$ of the nearest point $Z(i)$ and the distance $I_t$ between $Z(i)$ and $Z(j)$ and
STEP3: decide the trajectory $x(i+1)$ in one step future of the original time-series data.

## III. CORRELATION DIMENSION

The measurement of correlation dimension is employed to evaluate whether the time-series data are chaotic or not. The evaluation of the correlation dimension is pursued by checking whether the time-series data distribute in the less dimensional space than $m$-dimensional space, if the data is embedded in the $m$-dimensional space. At first, let us embed the time-series data into the $m$ dimensional space. Then, the procedure is written as follows:

(1) The procedure:
STEP1: draw the circle with radius $r$ at the center of the points which each embedded vector has.
STEP2: count how many points are included within the drawn circle and measure its number $C$.

**Figure 6.** $C(r) = ar^m$




When the radius is large, then the large number of points should be included in the circle with radius $r$. As $C$ is an increasing function of $r$, let us denote it as $C(r)$. If plotted points are distributed evenly in the $m$-dimensional space, the number of points included within the circle should increase proportionally to the area of the circle, as radius $r$ increases.

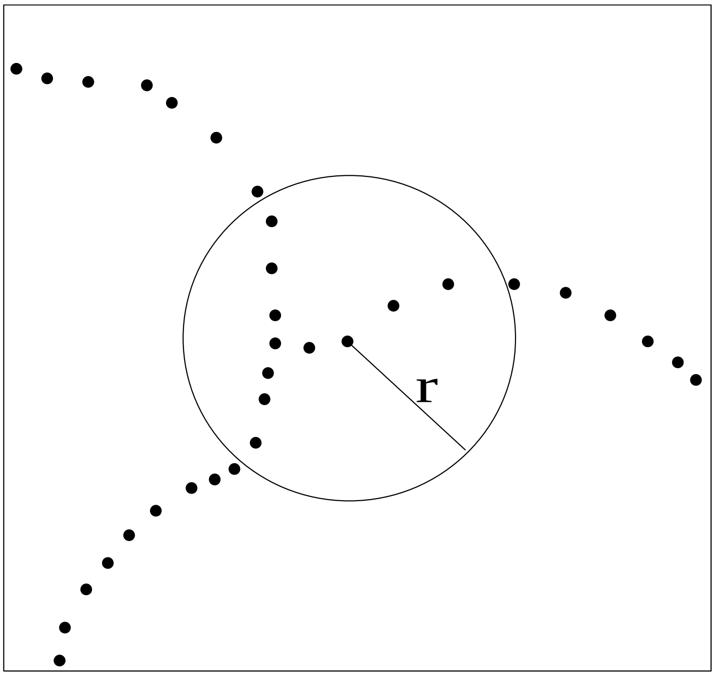
$$C(r) = ar^m \tag{4}$$

On the other hand, if the structure has any regularity, $C(r)$ should increase proportionally to the less value than m powered value as illustrated in figures 6 and 7.

$$C(r) = br^{(m-x)} \tag{5}$$

The value $(m-x)$ is named correlation dimension. In the case of random data, the regularity could not be found in the space even if the embedded dimension is increased. Therefore, the correlation dimension should increase even if the embedded dimension does. When the time-series data have the deterministic structure in the embedded space, the correlation dimension can not increase and should be matured at a certain value, even if the embedded dimension increases.

**Figure 7.** $C(r) = br^{m-x}$




## IV. WAVELET TRANSFORMATION

Fast Fourier Transform is a widely employed method to transform a signal into the portions of each frequencies. A sin function is employed as a base function. The sin function is an infinitive smooth function. Therefore, the information obtained by the Fast Fourier Transform does not include the local information such as the place and the frequency where and which frequency the original signals have.

On the other hand the wavelet transform employs a compact portion of a wavelet as a base function. Therefore, it is a time and frequency analysis such as it enables us to determine the signal using time and frequency.

The mother wavelet transform $(W_{\varphi} f)(b,a)$ of function $f(x)$ can be defined as follows:

$$(W_{\varphi} f)(b,a) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{|a|}} \overline{\psi\left(\frac{x-b}{a}\right)} f(x) dx \tag{6}$$

where $a$ is a scale of the wavelet, $b$ is a translate. $\overline{\psi(x)}$ is a conjunction of a complex number. It is also possible to recover the original signal $f(x)$ using wavelet transform.

That is, we can realize the inverse wavelet transform as follows:

$$f(x)=\frac{1}{C_\psi}\iint_R (W_\psi f)(b,a)\frac{1}{\sqrt{|a|}}\psi\left(\frac{x-b}{a}\right)\frac{\partial a\partial b}{a^2} \quad (7)$$

The wavelet transform is a useful method to know the characteristics of the signal but not an efficient one. It is because the signal has a minimum unit and the wavelet method expresses many-duplicated information's. This point can be resolved by discretizing a dimensional axis. Let us denote dimension as $(b,1/a)=(2-jk,2j)$, then the discrete wavelet transform can be rewritten as

$$d_k^{(j)}=2^j\int_{-\infty}^{\infty}\overline{\psi(2^j x-k)}f(x)dx \quad (8)$$

Inverse wavelet transform is

$$f(x)\sim\sum_j\sum_k d_k^{(j)}\psi(2^j x-k) \quad (9)$$

Let us denote the summation $\sum_k d_k^{(j)}\psi(2^j x-k)$ of the right term as

$$g_j(x)=\sum_k d_k^{(j)}\psi(2^j x-k) \quad (10)$$

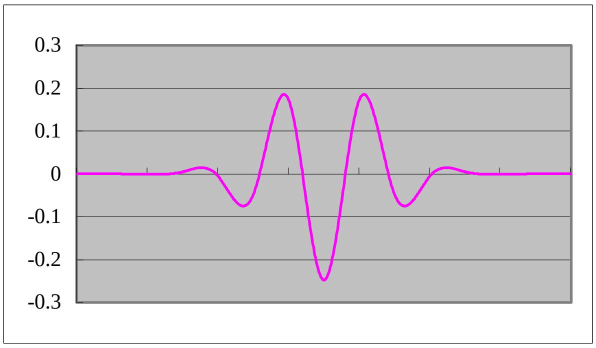
Then let us define $f_j(x)$ as

$$f_j(x)=g_{j-1}(x)+g_{j-2}(x)+\cdots \quad (11)$$

where an integer $j$ is named a level. If we can denote $f(x)$ as $f_0(x)$, then

$$f_0(x)=g_{-1}(x)+g_{-2}(x)+\cdots \quad (12)$$

**Figure 8 Spline4: Mother wavelet**




This equation illustrates that the function $f_0(x)$ is transformed into wavelet components $g_{-1}(x)$, $g_{-2}(x)$, ..., . It is required that the left side should be transformed uniquely into the right side and also the left side should be realized by composition from the right side components. They can be realized by using a mother wavelet $\psi$ as a base function. Function $f_j(x)$ can be rewritten using a recursive forms

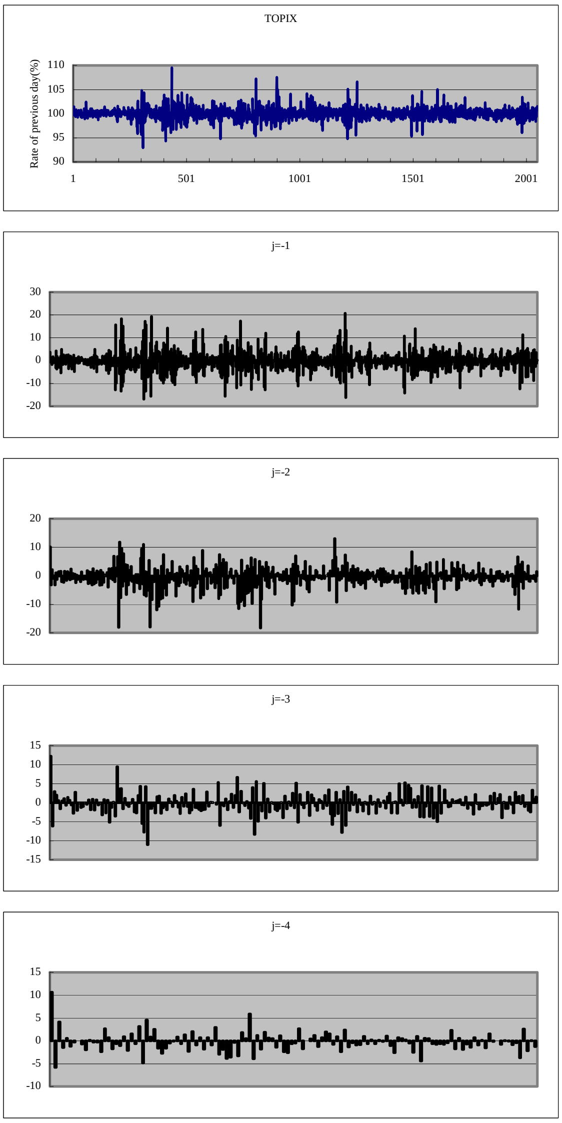
$$f_j(x) = g_{j-1}(x) + f_{j-1}(x) \qquad (13)$$

This equation means that the original signal $f_j(x)$ can be transformed into wavelet components $g_{j-1}(x)$ and $f_{j-1}(x)$. This equation enables us to decompose the original into the wavelet components step by step. This method is named multi-resolution signal decomposition.

## V. CORRELATION DIMENSION OF TRANSFORMED WAVELET COMPONENTS

Let us transform the time-series data into frequent components by Wavelet multi-resolve analysis. Spline4 shown in Fig.8 is employed as a mother wavelet function and the transformation was done until level 4. The time-series data analyzed is Tokyo stock average index TOPIX. The number of the data employed is 2048 samples from January 1991. Figure 9 shows the result obtained by the multi-resolve analysis. The first figure in Figure 9 is the original one. The smaller value j shows the lower frequency component.

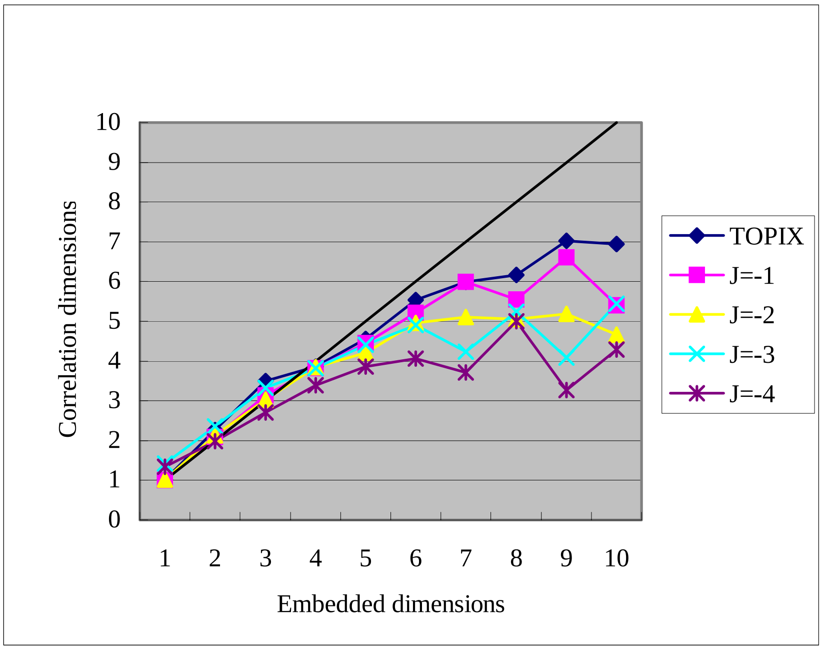
**Figure 9. Divided Time-series Data**




The result of Wavelet transformation shows that TOPIX has each frequency component smoothly.

We measured the correlation dimension of each component decomposed by Wavelet transform. The results are shown in Figure 10. The original TOPIX data shows that correlation dimension is matured at around 7. On the other hand, the wavelet component $j = -1$ is matured at around 6, the correlation dimension of the Wavelet component whose $j$ is less than or equal to -2 is matured around 4 to 5.

**Figure 10. Measurement of Correlation dimension about transformed component data and the original data**




Therefore, the results illustrate that the transformed components are more chaotic than the original TOPIX time-series data. The measurement of correlation dimensions results in that the component time-series data are more chaotic than the original time-series data. Let us forecast the short-term future using the original data and decomposed wavelet transformed component data and compare the preciseness between both results. The data are the same TOPIX data employed above. In this discussion the data were normalized into mean 0 and variance 1. The embedded dimensions are examined from dimensions 3 to 9. We measured the forecast errors about them. In the forecast, we employed remaining 100 data out of 2048 in the recent portion for the original examination and the transformed component examination. Figure 11 shows the results of the original and the wavelet transformed component time-series data. The vertical axis denotes error means and the horizontal axis shows embedded dimension.

The transformed component data shows drastically lower forecasting error than the original data. This shows that the component data is much more chaotic than the original TOPIX.

We could show that the wavelet transformed component time-series data are more chaotic than the original one both from the measurement of correlation dimension and from the forecasting error.

Let us examine the forecasting of the up and down movements of the price. This is to forecast the movement of the price, to which direction the tomorrow's price of a stock goes from the today's price. This forecasting is only done on the movement instead of on the value.

Let us check the prediction of up and down movements of the stock price based on the forecasted results. The up and down prediction was done for the movement of their following day's price using the TOPICS index data. The prediction is correct, if the direction of the movement is the same between the real and predicted movements. The percentage of the correct predictions is shown for the 100 trials.

As shown in Figure 12, the vertical axis is correct prediction rate and the horizontal axis is embedded dimension.

The component data by Wavelet transform shows better than the original time-series data. The transformed component of the original data is much better to predict the chaotic movement. In the case j=-4 the prediction is better than 70%. This rate is very high prediction.

Figure 11. The prediction error of transformed component time-series data

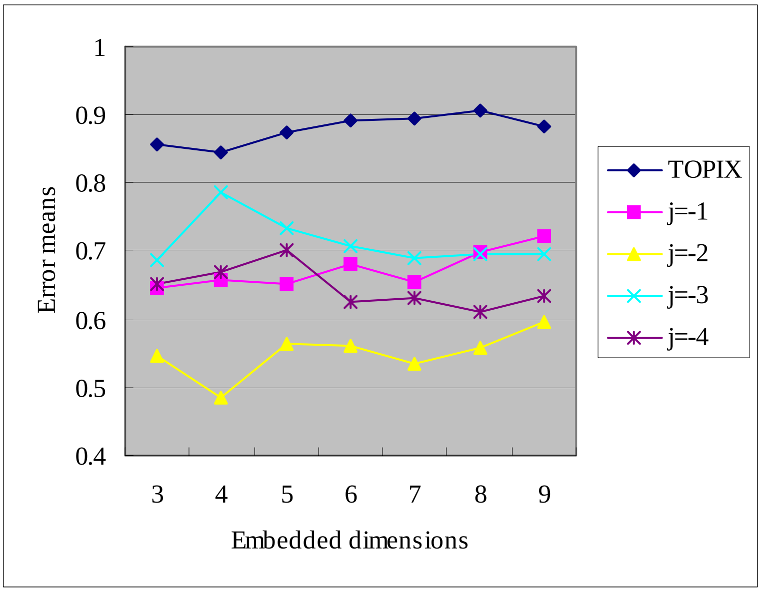
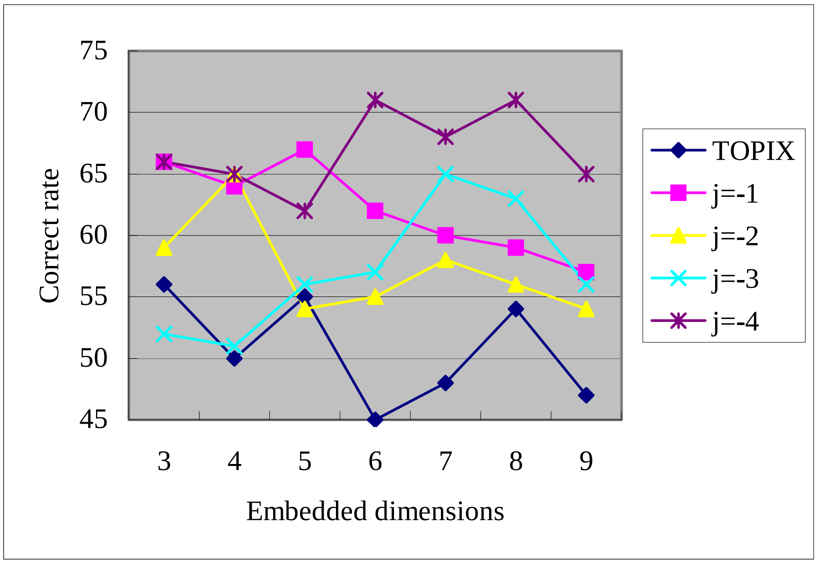



Figure 12. Correct rates on up and down prediction




## VI. CONCLUDING REMARKS

The objective of this paper is to realize the short-term forecasting based on Wavelet transform. Original time-series data are divided into components through Wavelet transform. The chaotic short-term forecasting method is applied to the transformed component decomposed by Wavelet transform.

It should be noted that even if the given time-series data have lower chaotic characteristics, we can derive the highly chaotic structure in the components obtained by Wavelet transform. This means that even if the given data is less chaotic, we can derive partially chaotic component from the original data through the Wavelet transform.

We discussed about the measurement of the correlation dimension, forecasting error and the correct prediction rate by the comparison between the original data and the component of the Wavelet transform using TOPIX data which is an index of the Japanese stock market. The correlation dimension of the component is lower than one of the original data. TOPIX data obtained lower correlation dimension than individual stock-price data. The lower correlation dimension means higher chaotic characteristics. TOPIX showed that the divided component data obtained by Wavelet transform proved higher prediction rate up to 70% than the original data. This shows the wavelet transform can abstract the chaotic component well from the original data.

## REFERENCES

[1] Matsuba, I., Chaos and forecast (1992). Mathematical Sciences, No.348, pp.64-69, in Japanese.

[2]Nagashima, T., Nagai, Y., Ogiwara, T., and Tsuchiya T., (1990). Time series data analysis and Chaos, Sensing Instrument Control Engineering, Vol. 29, No. 9, in Japanese.

[3] Matsumoto, Y., and Watada, J., (1998). Short-tem Prediction by Chaos Method of Embedding Related Data at Same Time, *Journal of Japan Industrial Management Association*, Vol. 49, No. 4, pp. 209-217, in Japanese.

[4]Matsumoto, Y., and Watada, J., (1998) Application of Chaotic Short-term Forecast to Economics and Business Problem, Vietnam-Japan Bilateral Symposium on Fuzzy Systems and Applications, pp. 219-225, Vietnam.

[5] Matsumoto, Y., and Watada, J., (2001). Chaotic Short-term Forecasting on Up & Down Movements of Stock Prices, 2nd International Symposium on Advanced Intelligent Systems, pp. 287-291, Korea.

[6] C.K. Chui, C. K. (1992). Introduction to wavelets, Academic Press, New York.

[7] Brock, W. A. (1986). Distinguishing Random and Deterministic Systems: A Bridged Version, *Journal of Economic Theory*, 40, pp.168-195.

[8] J.A. Scheinkman, J. A. and Blake LeBaron (1989). Nonlinear Dynamics and Stock Returns, Journal of Business, Vol.62, No.3, pp.311-337.

[9] Serizawa, H. (1993). Phenomenon knowledge of chaos, Tokyo Books, in Japanese.

[10]Takens, F., (1981). Detecting Strange Attractors in Turbulence, in Dynamical Systems and Turbulence, ed. by D.A.Rand and L.S.Young, Lecture Notes in Mathematics, vol.898 (Springer-Verlag, Berlin), pp.366-381.